Computational Social Choice

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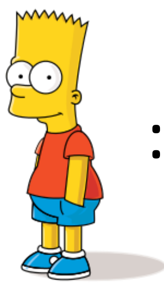
Rank Aggregation

- Rank aggregation (also known as voting) is the problem of aggregating several ordered lists of alternatives
- Input:
  - a set of alternatives (candidates) $C = \{c_1, \ldots, c_m\}$
  - a set of voters $V = \{1, \ldots, n\}$
  - for each voter, a total order (ranking) over C
- Output:
  - a winner
  - a set of winners
  - a total ranking of the alternatives

Rank Aggregation: Examples

- What movie should the Simpson family watch?

: Frozen > Paddington > Minions



: Paddington > Minions > Frozen

: Minions > Paddington > Frozen

Rank Aggregation: Examples

- Which PhD applicant should the algorithmic game theory group at Oxford accept?
  - Paul: X > Y > Z
  - Elias: Y > X > Z
  - Edith: Z > Y > X

Rank Aggregation: Examples

- United Kingdom (specific precinct)
  - candidates: Conservatives (C), Labour (L), Liberal Democrats (LD)
  - 60 000 voters
  - 25 000 voters prefer C to LD to L: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L

Example: Competition for a Fellowship

- Candidates: 50 students
- Voters: 15 panel members
  - each panel member has a ranking of the candidates
    (or perhaps top 10 candidates)
- Goal: select 10 students who will get a fellowship
- Asking each panel member to vote for her favorite candidate is not appropriate:
  - at most 7 students can get 2 or more votes

Example: Ranking of the Universities

- A panel of experts is supposed to rank UK universities
  - Expert 1: Cambridge > Oxford > UCL > Imperial
  - Expert 2: Oxford > Cambridge > Imperial > UCL
  - Expert 3: UCL > Cambridge > Oxford > Imperial
  - Expert 4: Oxford > Imperial > Cambridge > UCL
  - Expert 5: Imperial > Cambridge > UCL > Oxford
- Goal: produce a total ranking of 4 universities

Example: Ranking of the Universities

- A panel is supposed to rank UK universities
- Rankings are based on 5 different criteria:
  - reputation ranking
  - grant income
  - student satisfaction
  - number of research papers published
  - average salary after graduation
- Rankings:
  - criterion 1: Cambridge > Oxford > Imperial > UCL
  - criterion 2: Oxford > Cambridge > UCL > Imperial
  - criterion 3: UCL > Cambridge > Oxford > Imperial
  - criterion 4: Oxford > Imperial > Cambridge > UCL
  - criterion 5: Imperial > Cambridge > UCL > Oxford
- Should all criteria have the same weight?

Part 1:
the zoo of voting rules

Single-Winner Rules: Plurality

- Plurality:
  - each voter names his favorite candidate
  - candidates with the largest number of votes win
  - if two or more candidates get the highest score, the winner is chosen using some tie-breaking rule
- For 2 candidates,
  Plurality selects the majority winner

Political Voting

- United Kingdom (specific precinct)
  - candidates: Conservatives (C), Labour (L), Liberal Democrats (LD)
  - 60 000 voters
  - 25 000 voters prefer C to LD to L: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
  - Plurality outcome: C wins with 25000 votes

Single-Winner Rules: Plurality

- Plurality is obviously the best voting rule if there are only 2 candidates
- However, for 3 candidates it may behave in an undesirable way
  - the majority of voters may prefer some other alternative to the current winner
  - voters have an incentive to vote non-truthfully

Plurality: Example Revisited

- United Kingdom elections:
  - 25 000 voters: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
- Outcome under Plurality:
  - C wins with 25000 votes
- Undesirable properties:
  - 31 000 voters prefer L to C, 35 000 voters prefer LD to C
  - the voters with ranking LD > L > C
    would be better off voting L

Two-Round Elections

1. All voters vote for their favorite candidate
2. All but the two highest-scoring candidates are eliminated
3. The voters are asked to vote again over the remaining candidates

This rule is known as Plurality with Runoff; used in France for presidential elections

Plurality With Runoff: Example

- United Kingdom elections:
  - 25 000 voters: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
- 1st round: C: 25 000, L: 20 000, LD: 15 000
  - noone has more than 30 000 votes, so LD is eliminated
- 2nd round: C: 25 000+4 000, L: 20 000 + 11 000
  - L gets the majority of votes, so it wins

Multi-Round Elections

1. All voters vote for their favorite candidate
2. If some candidate gets more than 50% of the votes, he is declared the winner
3. Otherwise, the candidate with the smallest number of votes is eliminated
4. The voters are asked to vote again over the remaining candidates
5. The process repeats until some candidate gets a majority of votes

Single Transferable Vote

- Multi-round elections often produce a more appealing outcome than Plurality
- However, they are hard to implement:
  - voters have to come to voting booths many times
- Single Transferable Vote: an implementation of multi-round elections in a single round of voting
  - each voter submits a total ranking of candidates
  - the election authority simulates multi-winner elections based on the information in the ballots (assuming that all voters always vote for their most preferred available candidate)
- UK had a referendum of switching to STV on May 5th, 2011 - but the decision was “no”

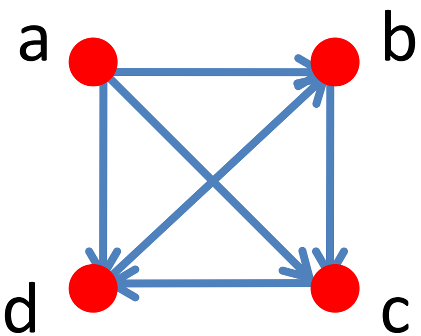
How Good are Plurality With Runoff and STV?

- United Kingdom elections:
  - 25 000 voters: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
- Plurality chooses C, STV chooses L
- Yet, 40 000 voters prefer LD to L and 35 000 voters prefer LD to C
- Under both Plurality and STV, more than 50% of voters would have preferred a different candidate
- Under STV, the voters who rank C first would be better off voting for LD

Condorcet Winners

- Suppose that each of the n voters has a ranking of all m candidates
- Definition: a candidate c wins a pairwise election against a candidate d if more than half of the voters rank c above d
- A candidate is said to be a Condorcet winner if he wins in all pairwise elections he participates in

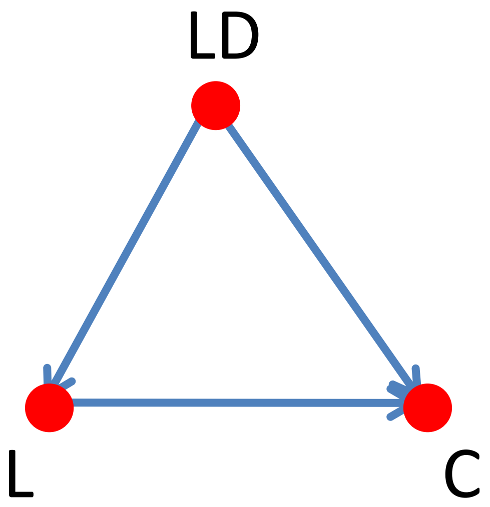
b	c	d	a	d
a	a	a	b	c
c	d	b	c	b
d	b	c	d	a



a is the Condorcet winner

Condorcet Consistency

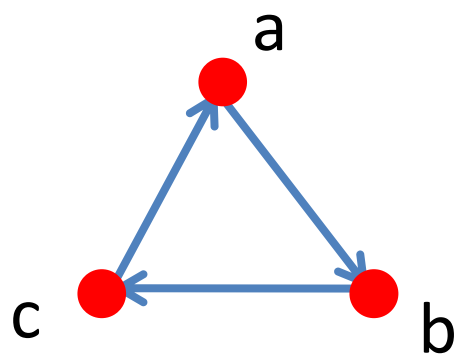
- A voting rule is said to be Condorcet-consistent if it selects the Condorcet winner whenever it exists
- United Kingdom elections:
  - 25 000 voters: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
- Plurality chooses C, STV chooses L
- LD is the Condorcet winner
  - even though it has the smallest number of voters who rank it first
- Hence, neither Plurality nor STV are Condorcet-consistent



Do Elections Always Have Condorcet Winners?

- 2 voters rank a above b
- 2 voters rank b above c
- 2 voters rank c above a
- No Condorcet winner!

a	c	b
b	a	c
c	b	a



- Definition: G is a pairwise majority graph for an election E with a candidate set C if its vertex set is C and there is an edge from a to b iff majority of voters prefer a to b
- Theorem: any directed graph with no 2-cycles can arise as a pairwise majority graph

Condorcet-Consistent Rules: Copeland

- A Condorcet-consistent rule must elect a Condorcet winner when one exists
  - how can we extend this principle if there is no Condorcet winner?
- Copeland rule: each candidate gets
  - 1 point for each pairwise election he wins
  - 0.5 points for each pairwise election he ties
  - the candidate with the largest number of points wins
- In an m-candidate election, if a Condorcet winner exists, he gets m-1 point, all other candidates get at most m-2 points

Condorcet-Consistent Rules: Maximin

- Maximin rule: the score of each candidate is the number of votes he gets in his worst pairwise election
  - the candidate with the highest score wins
- In an n-voter election, if a Condorcet winner exists
  - his Maximin score is greater than $n/2$,
  - everyone else's Maximin score is less than $n/2$

Condorcet-Consistent Rules: Dodgson

- Dodgson score of a candidate X: a number of swaps of adjacent candidates needed to make X the Condorcet winner
  - NP-hard to compute
- Dodgson winner(s): the candidate(s) with the smallest Dodgson score

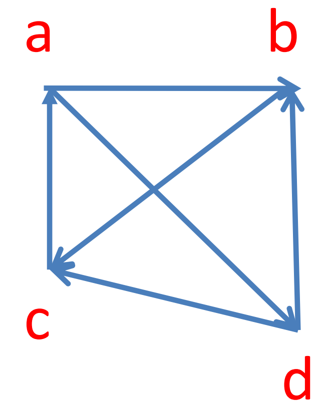
a d b c

a d c b

b d c a

c a d b

b d c a



Scoring Rules

- Condorcet-consistent rules are hard to explain to voters
  - implementation is non-trivial
- Alternative: scoring rules
- A scoring rule for an election with m candidates is given by a vector $(s_1, \ldots, s_m)$, $s_1 \geq \ldots \geq s_m$
  - each candidate gets s_i points from each voter who ranks him i-th
  - candidate with the maximum number of points wins
- Plurality is a scoring rule with score vector $(1, 0, \ldots, 0)$
- Borda: $(m-1, m-2, \ldots, 2, 1, 0)$
- k-approval: $(\underbrace{1, \ldots, 1}_{k}, 0, \ldots, 0)$
  - equivalent to allowing voters to vote for k candidates

Competition for a Fellowship Revisited

- Candidates: 50 students
- Voters: 15 panel members
  - each panel member has a ranking of the candidates (or perhaps top 10 candidates)
- Goal: select 10 students who will get a fellowship
- 10-approval (aka Bloc):
  - each voter is asked to vote for top 10 candidates
- Truncated Borda:
  - each voter is asked to identify top 10 candidates, and order them
  - each student gets 11 - i points from each voter who ranks him in position i
- In either case, students with top 10 total scores win

Scoring Rules

- United Kingdom elections:
  - 25 000 voters: C > LD > L
  - 20 000 voters: L > LD > C
  - 11 000 voters: LD > L > C
  - 4 000 voters: LD > C > L
- Plurality: C wins with 25 000 points
- Borda:
  - C gets 2 x 25 000 + 1 x 4 000 = 54 000 points
  - L gets 2 x 20 000 + 1 x 11 000 = 51 000 points
  - LD gets 1 x 45 000 + 2 x 15 000 = 75 000 points
- 2-approval:
  - C: 29 000, L: 31 000, LD: 60 000

Scoring Rules: Pro and Contra

- Scoring rules are easy to understand and implement
- They take into account preferences other than just the voter's top choice
- However, no scoring rule is Condorcet-consistent
  - Borda: a b c d e | a b e d c | b c d e a
  - a is the Condorcet winner, yet a gets 8 points, while b gets 10
- Borda rule is very easy to manipulate:
  - 3 voters: a > b > c > d > e
  - 1 voter: b > a > c > d > e

  a: 15, b: 13
  - if the last voter, who prefers b, votes b > c > d > e > a, a loses 3 points, so b wins

Bucklin's rule

- How do we choose k for k-approval?
- One possible answer: adaptively
- Let k* be the smallest value of k such that there is a candidate ranked in top k positions by more than n/2 voters
- Bucklin rule: output all k*-approval winners
- Alternative interpretation:
  - for k =1, …., m do
  - ask each voter to name their top k candidates
  - stop when some candidate is named by a majority
  - report all such candidates

Schulze's Rule

- Consider the weighted majority graph
  - the weight of the edge AB is the number of voters who prefer A to B
  - only keep edges whose weight is $\geq n/2$
- Strength of a path from A to B:
  min weight along that path
- p[A, B]: strength of the strongest path from A to B
- A is a winner if p[A, B] $\geq$ p[B, A] for all B
  - always exists

Ties?

- All rules defined so far may produce multiple winners
- In a sense, this is unavoidable
  - suppose input election contains a single copy of each of the $m!$ permutations of candidates
- Tie-breaking:
  - lexicographic (based on a candidate order)
  - randomized
    - uniform over top-scoring candidates
    - pick a random voter, ask her to break the tie

Rankings: Social Welfare Functions

- Score-based rules can be used to produce rankings: order candidates by score
  - not just scoring functions, but also Copeland, Maximin, etc.

- Kemeny rule:
  - for two votes u,v, let $d(u,v) = \# \{(A,B): A >_u B, B >_v A\}$
  - find a ranking that minimizes the total distance to votes

Ranking of the Universities: Borda

- A panel of experts is supposed to rank UK universities
  - Expert 1: Cambridge > Oxford > UCL > Imperial
  - Expert 2: Oxford > Cambridge > Imperial > UCL
  - Expert 3: UCL > Cambridge > Oxford > Imperial
  - Expert 4: Oxford > Imperial > Cambridge > UCL
  - Expert 5: IMperial > Cambridge > UCL > Oxford
- Goal: produce a total ranking of 4 universities
- Borda rule:
  - each university gets 4-i points from each expert who ranks it in position i
  - Cambridge: 10, Oxford: 9, UCL: 5, Imperial: 6

Ranking of the Universities: Kemeny

- A panel of experts is supposed to rank UK universities
  - Expert 1: Cambridge > Oxford > UCL > Imperial
  - Expert 2: Oxford > Cambridge > Imperial > UCL
  - Expert 3: UCL > Cambridge > Oxford > Imperial
  - Expert 4: Oxford > Imperial > Cambridge > UCL
  - Expert 5: Imperial > Cambridge > UCL > Oxford
- Goal: produce a total ranking of 4 universities
- Kemeny rule:
  - need to score each of the 24 possible rankings
  - e.g., Oxford > Cambridge > UCL > Imperial scores 5+5+3+1

Complexity of Winner Determination

- Can we efficiently compute the outcome of a voting rule?
  - poly-time algorithms: scoring rules, Copeland, Maximin, Schulze
  - NP-hard: Dodgson, Kemeny
  - it’s complicated: STV
    - we can run STV breaking ties in some way and find some winner
    - it is NP-hard to decide whether a given candidate is a winner for some way of breaking ties

Part 2:
Justifying Voting Rules

Desirable Properties of Voting Rules

- Anonymity: all voters are treated in the same way

  +: all

- Neutrality: all candidates are treated in the same way

  +: all (ties?)

- Condorcet consistency

  +: Copeland, Maximin, Dodgson, Schulze

  -: Plurality, Plurality with Runoff, STV, Borda

Criteria for Voting Rules: Single-Winner Elections

- Consistency: consider two elections with disjoint sets of voters over the same set of candidates. If c wins in both elections, he should also win when we merge these two elections
  - +: scoring rules
  - -: (nearly) everything ese
- Pareto efficiency: if all voters rank a above b, b should not win
  - +: all

Criteria for Voting Rules: Single-Winner Elections

- Monotonicity: if c wins, and some voter moves c higher in her ranking, without changing the order of other candidates, then c still wins

  +: Plurality, Copeland, Maximin, Borda, Schulze

  -: Plurality with Runoff, STV

  Example (STV):

A moves to the top in the first 2 votes

7 votes B > C > A
7 votes A > B > C
6 votes C > A > B

C drops out
A wins

→

5 votes B > C > A
9 votes A > B > C
6 votes C > A > B

B drops out
C wins

Criteria for Voting Rules: Rankings

- Pareto efficiency: if all voters rank a above b, in the final ranking a should appear above b
- Monotonicity: if some voter moves c up in their ranking, in the overall ranking c goes up
- Independence of irrelevant alternatives (IIA): if a is ranked above b in the current election, and we permute the candidates in each vote without changing the relative order of a and b, then a should be ranked above b in the resulting election

Dictatorship

- There is a very simple rule that produces a ranking of alternatives and satisfies all of our criteria: dictatorship
- This rule simply copies the ranking of some fixed voter
- Satisfies monotonicity, Pareto-optimality, IIA
- Truthful voting is a dominant strategy
- Is usually not an acceptable voting rule for obvious reasons

Arrow’s Theorem [1951]

- Suppose there are at least 3 candidates
- Then any voting rule that produces a ranking of all candidates and is simultaneously:
  - Pareto efficient and
  - independent of irrelevant alternatives

is a dictatorship

“There is no perfect voting rule”

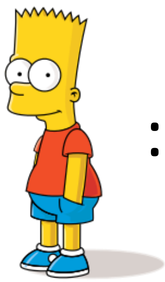
Gibbard-Satterthwaite Theorem

- Suppose there are at least 3 candidates. Then for any voting rule that is not a dictatorship there exists a list of voters' preferences such that some voter v has an incentive to vote non-truthfully
  - v can change his vote so that
    the winner is a candidate
    that v ranks higher than the original winner
- No voting rule is resistant to manipulative behavior!

Voting as Preference Aggregation

- What movie should the Simpson family watch?

: Frozen > Paddington > Minions



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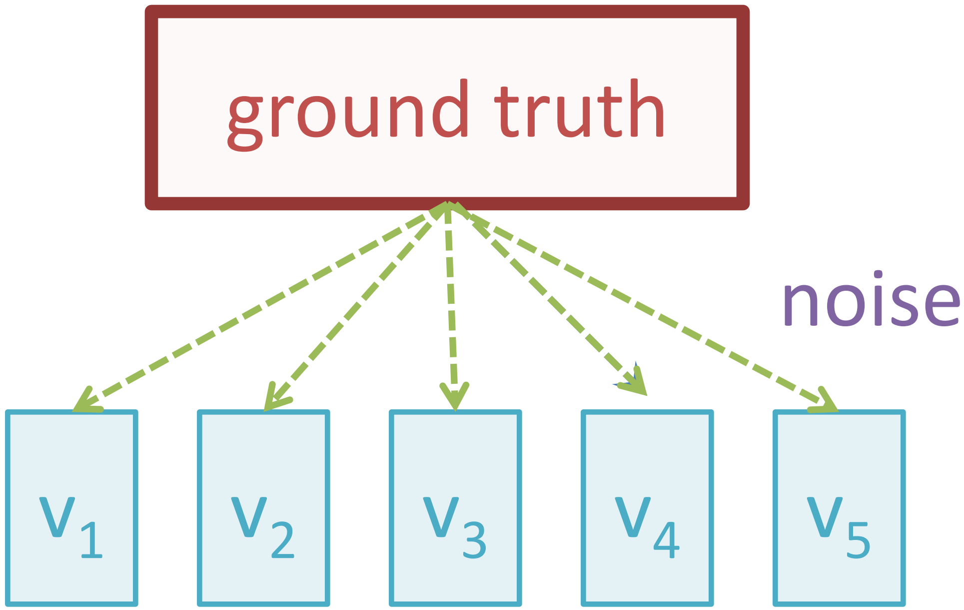
Voting as a Way to Uncover Truth

- Which cleaning company should we hire?
  - Adam: A > B > C
  - Ben: C > B > A
  - Charlie: B > C > A

- Which PhD applicant should we accept?
  - Paul: X > Y > Z
  - Elias: Y > X > Z
  - Edith: Z > Y > X

- Medieval church elections
- Crowdsourcing

Voting as Maximum Likelihood Estimation



Which true state of the world is most likely to generate the observed votes?

History

- Marquis de Condorcet (1785), Essai sur l'application de l'analyse a la probabilité des décisions rendues a la pluralité des voix
- …
- H. Peyton Young (1988), Condorcet's theory of voting,
  Am. Pol. Sci. Review
- …
- Elkind, Shah (2014), Choosing the most probable without eliminating the irrational: voting on intransitive domains, *UAI'14*

Condorcet-Young-Mallows Model

- m alternatives, n voters: $V = (v_1, \ldots, v_n)$
- Ground truth = ranking of the alternatives
- Votes = rankings of the alternatives
- Noise:
  - fix $½ < p < 1$
  - ground truth: u
  - each vote is an outcome of the following process:
    - pick a fresh pair of alternatives a, b; assume $a >_u b$
    - rank them as $a > b$ w.p. p and as $b > a$ w.p. $1-p$
    - if this produces a cycle, restart

Most Likely Ranking [Young'88]

- Kemeny distance: $d(u, v) = |\{(a, b): a >_u b, b >_v a\}|$
- $\phi = p/(1-p)$
- $Pr[v] \sim p^{m(m-1)/2 - d(u, v)} (1-p)^{d(u, v)}$
- $Pr[V] = Pr[v_1] \times \ldots \times Pr[v_n] \sim \phi^{-\Sigma_i d(u, v_i)}$
- $Pr[V] \sim \phi^{-d(u, V)}$
- Most likely ranking: one that minimizes the total distance to votes
  - Kemeny's rule

Rankings vs. Winners

- Finding the most likely ranking: Kemeny's rule
- Finding the most likely winner?
- $s_R(a)$: cumulative likelihood of rankings where a is ranked first
- $s_R(a) = \sum_{u:\, top(u)=a} \phi^{-d(u, V)}$
- Which a maximizes $s_R(a)$?

Most Likely Winner [Y'88, PRS'12]

- $s_R(a) = \Sigma_{u:\, top(u)=a} \phi^{-d(u, V)}$
- $s_R(a)$: sum of (m-1)! non-positive powers of ϕ
- $p \rightarrow 1$, $\phi = p/(1-p) \rightarrow \infty$ (low noise):
  - the set of most likely winners is a subset of Kemeny winners
- $p \rightarrow 1/2$, $\phi = p/(1-p) \rightarrow 1$ (high noise):
  - the set of most likely winners is a subset of Borda winners

Part 3:
Domain Restrictions

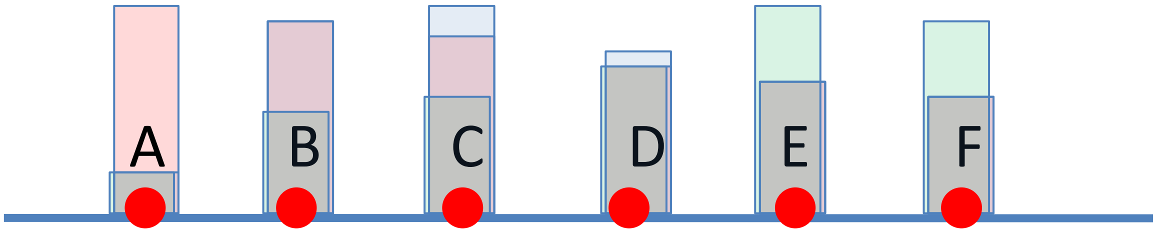
Difficulties

- Problem:
  with no assumption on preference structure
  - majority cycles may occur
  - all voting rules are manipulable
  - computing outcomes of some voting rules is NP-hard
- Solution: restrict the preference domain

A	B	C
B	C	A
C	A	B
D	D	D

Single-Peaked Preferences

- Definition: a vote v is single-peaked (SP) wrt an ordering < of candidates (axis) if it holds that:
  - if top(v) < D < E, v prefers D to E
  - if A < B < top(v), v prefers B to A
- Example:
  - voter 1: C > B > D > E > F > A
  - voter 2: A > B > C > D > E > F
  - voter 3: E > F > D > C > B > A



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Example: Temperature

- Perfect water temperature?

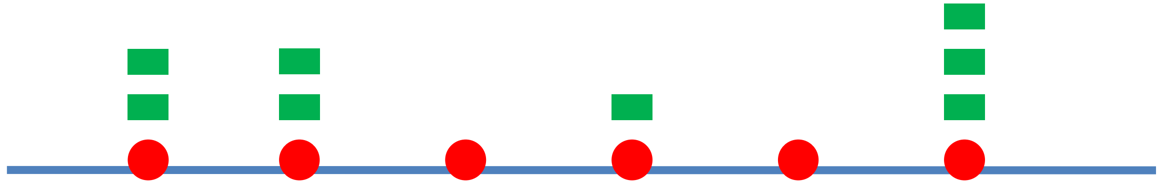
+16 +20 +23 +25 +27 +30

SP Preferences: Transitivity

- Theorem: in single-peaked elections with an odd number of voters the majority relation is transitive
  - if more than n/2 voters prefer a to b and more than n/2 voters prefer b to c then more than n/2 voters prefer a to c
- Lemma: each single-peaked election with an odd number of voters has a Condorcet winner (CW)
- Proof of the theorem (assuming the lemma):
  - by the lemma, there is a CW, say a
  - delete a from all votes; the profile remains SP
  - use induction

SP Preferences: Condorcet Winners

- Lemma: in single-peaked elections with an odd number of voters there exists a Condorcet winner (CW))
  - ask each voter v to vote for one candidate
    - let C(v) denote the vote of voter v
  - order voters by C(v), breaking ties arbitrarily
  - if we have $n = 2k+1$ voters, $top(v_{k+1})$ is a CW
  - even n: if we have $n = 2k$ voters, all candidates between $top(v_k)$ and $top(v_{k+1})$ are weak CWs

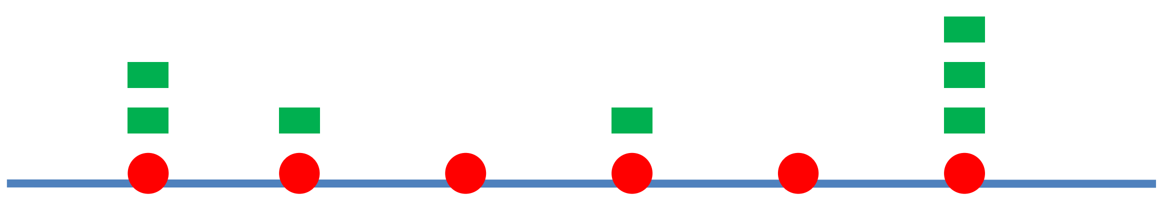


Transitivity: Consequences

- Theorem: in a single-peaked election with an odd number of voters the winning ranking under the Kemeny rule can be computed in polynomial time
  - Lemma: if the majority relation is transitive, the Kemeny ranking coincides with the majority relation.

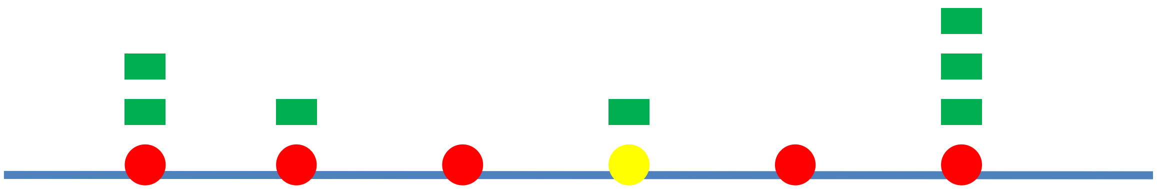
SP Preferences: Circumventing Gibbard-Satterthwaite

- Suppose we have $n = 2k+1$ voters
- Median voter rule:
  - consider an election that is single-peaked wrt $<$
  - ask each voter v to vote for one candidate
    - let $C(v)$ denote the vote of voter v
  - order voters by $C(v)$, breaking ties arbitrarily
  - output $C^* = C(v_{k+1})$



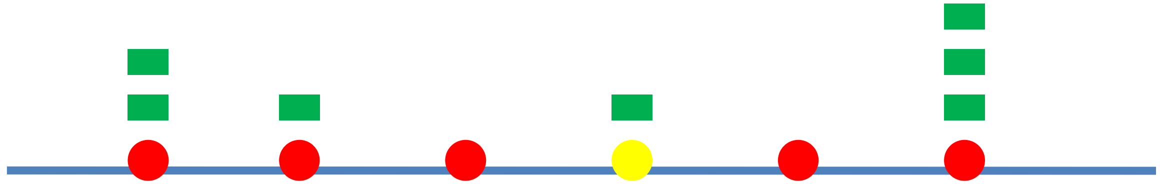
SP Preferences: Median Is Truthful

- Theorem: under the median voter rule, it is a dominant strategy to vote for one's top choice
- Consider a voter v_i in our order
  - $i = k+1$: v_i gets his most preferred outcome
  - $i < k+1$ ($i > k+1$ is symmetric):
    - if v_i votes C, $C \leq C^*$, v_{k+1} remains the median voter, so the outcome does not change



SP Preferences: Median is Truthful

- Theorem: under the median voter rule, it is a dominant strategy to vote for one's top choice
- Consider a voter v_i in our order
  - $i = k+1$: v_i gets his most preferred outcome
  - $i < k+1$ ($i > k+1$ is symmetric):
    - if v_i votes C, $C \leq C^*$, v_{k+1} remains the median voter, so the outcome does not change
    - if v_i votes C, $C^* < C$, either v_i (with his new vote) or v_{k+2} becomes the median voter, so the outcome gets worse for v_i



Single-Crossing Preferences

Definition: a profile is single-crossing (SC) wrt an ordering of voters ($v_1, \ldots, v_n$) if for each pair of candidates A, B there exists an $i \in \{0, \ldots, n\}$ such that voters $v_1, \ldots, v_i$ prefer A to B, and voters $v_{i+1}, \ldots, v_n$ prefer B to A

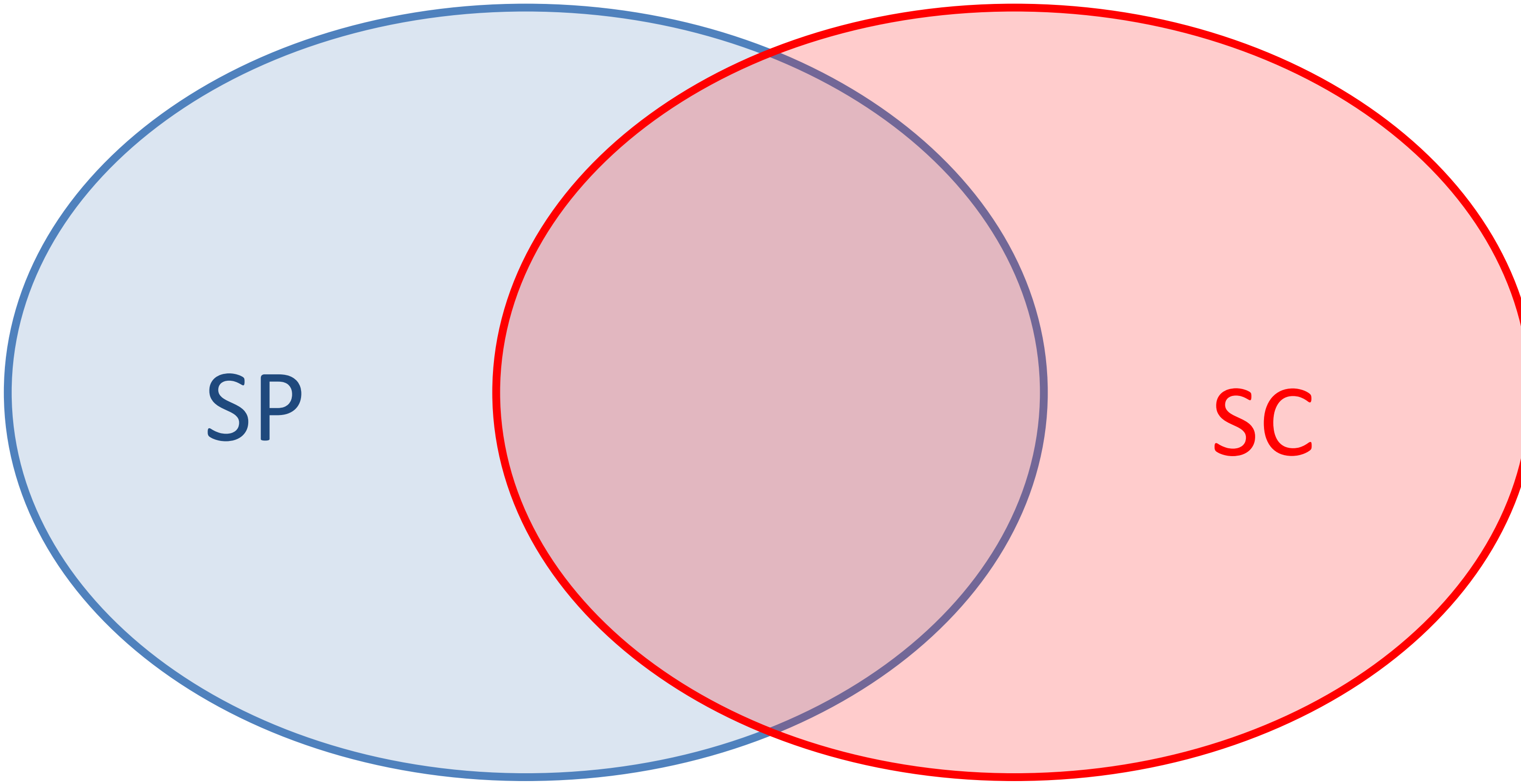
A	B	B	C	C	C	D
B	A	C	B	B	D	C
C	C	A	A	D	B	B
D	D	D	D	A	A	A

SC Preferences: Majority is Transitive

- Claim: in single-crossing elections, the majority relation is (weakly) transitive
  - we will prove the claim for $n=2k+1$ voters
  - consider the ranking of voter v_{k+1}
  - if v_{k+1} prefers B to A, so do $\geq k$ other voters
- Claim: the SC order of voters is essentially unique

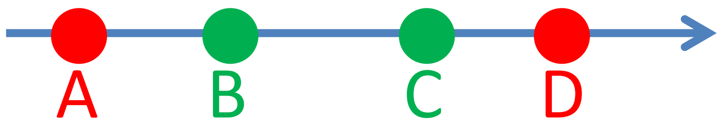
A	B	B	C	C	C	D
B	A	C	B	B	D	C
C	C	A	A	D	B	B
D	D	D	D	A	A	A

???


SP
SC

Single-Peaked Profile That Is Not Single-Crossing

B	B	C	C
C	C	B	B
A	D	A	D
D	A	D	A



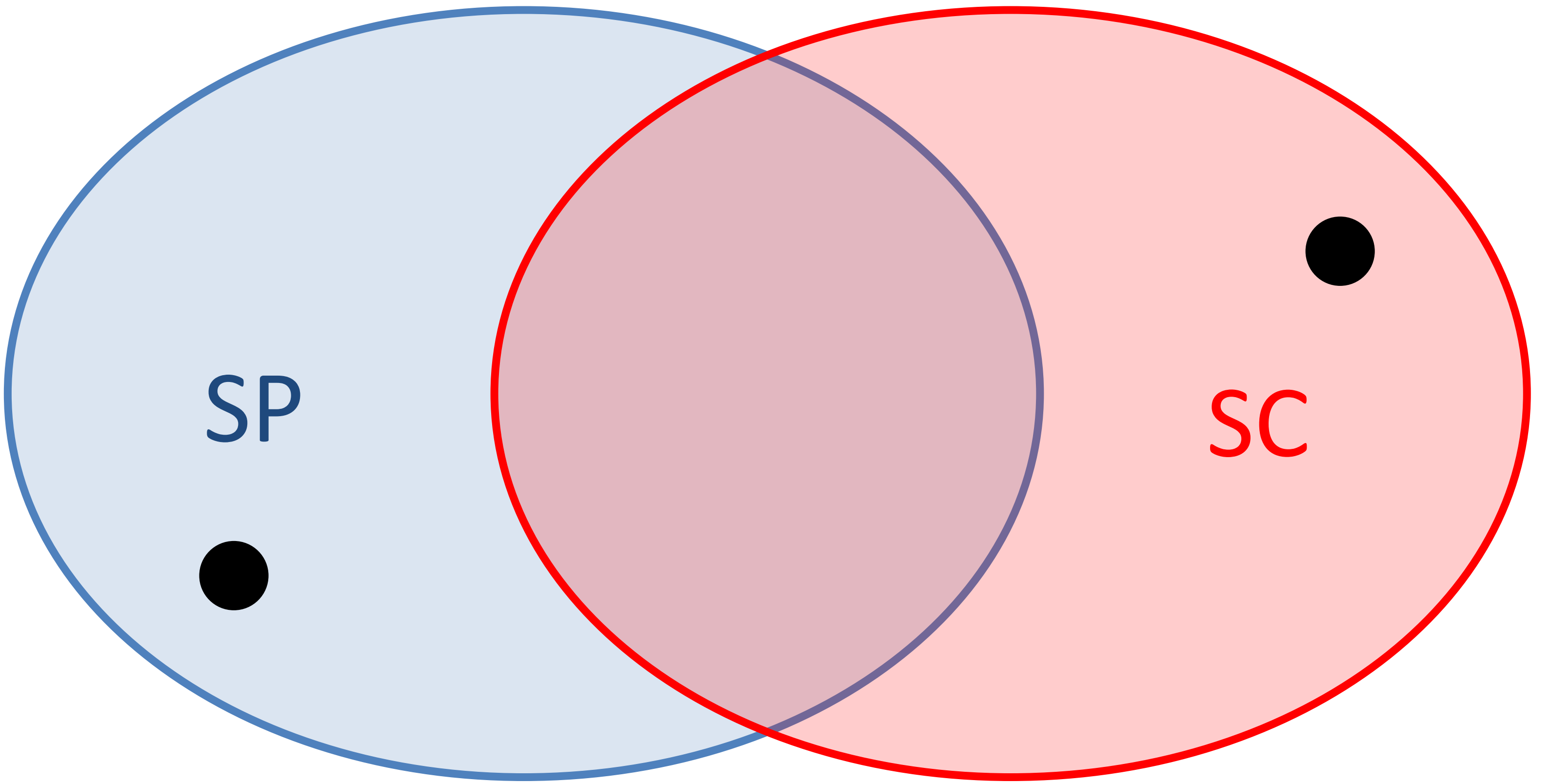
- v_1 and v_2 have to be adjacent (because of B, C)
- v_3 and v_4 have to be adjacent (because of B, C)
- v_1 and v_3 have to be adjacent (because of A, D)
- v_2 and v_4 have to be adjacent (because of A, D)

a contradiction

Single-Crossing Profile That Is Not Single-Peaked

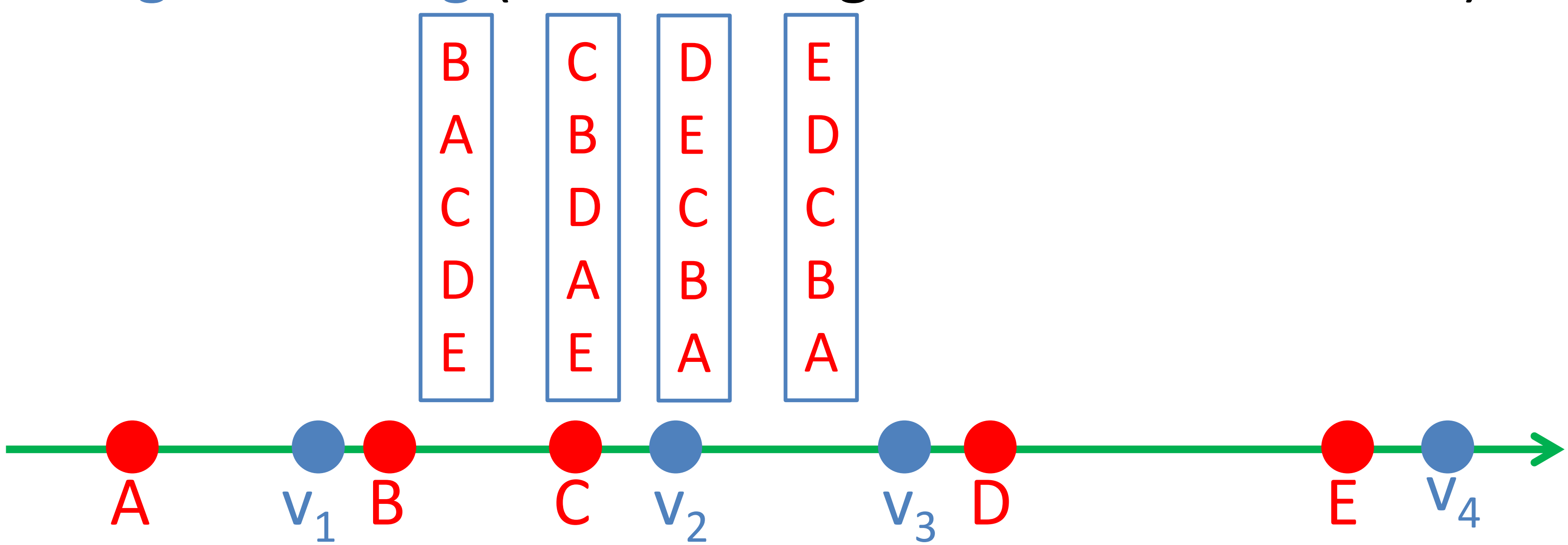
1	n	n		n	n
2	1	n-1		n-1	n-1
…	2	1		n-2	n-2
…	…	2	…	…	…
…	…	…		…	…
n-2	…	…		…	…
n-1	n-2	…		1	2
n	n-1	n-2		2	1

Each candidate is ranked last exactly once


SP
SC

1D-Euclidean Preferences

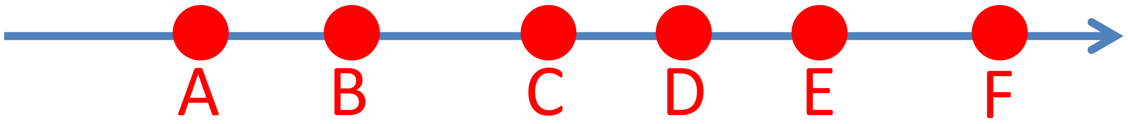
- Both voters and candidates are points in $\mathbb{R}$
- v prefers A to B if |v - A| < |v - B|
- <u>Observation</u>: 1D-Euclidean preferences are
  - single-peaked (wrt ordering of candidates on the line)
  - single-crossing (wrt ordering of voters on the line)



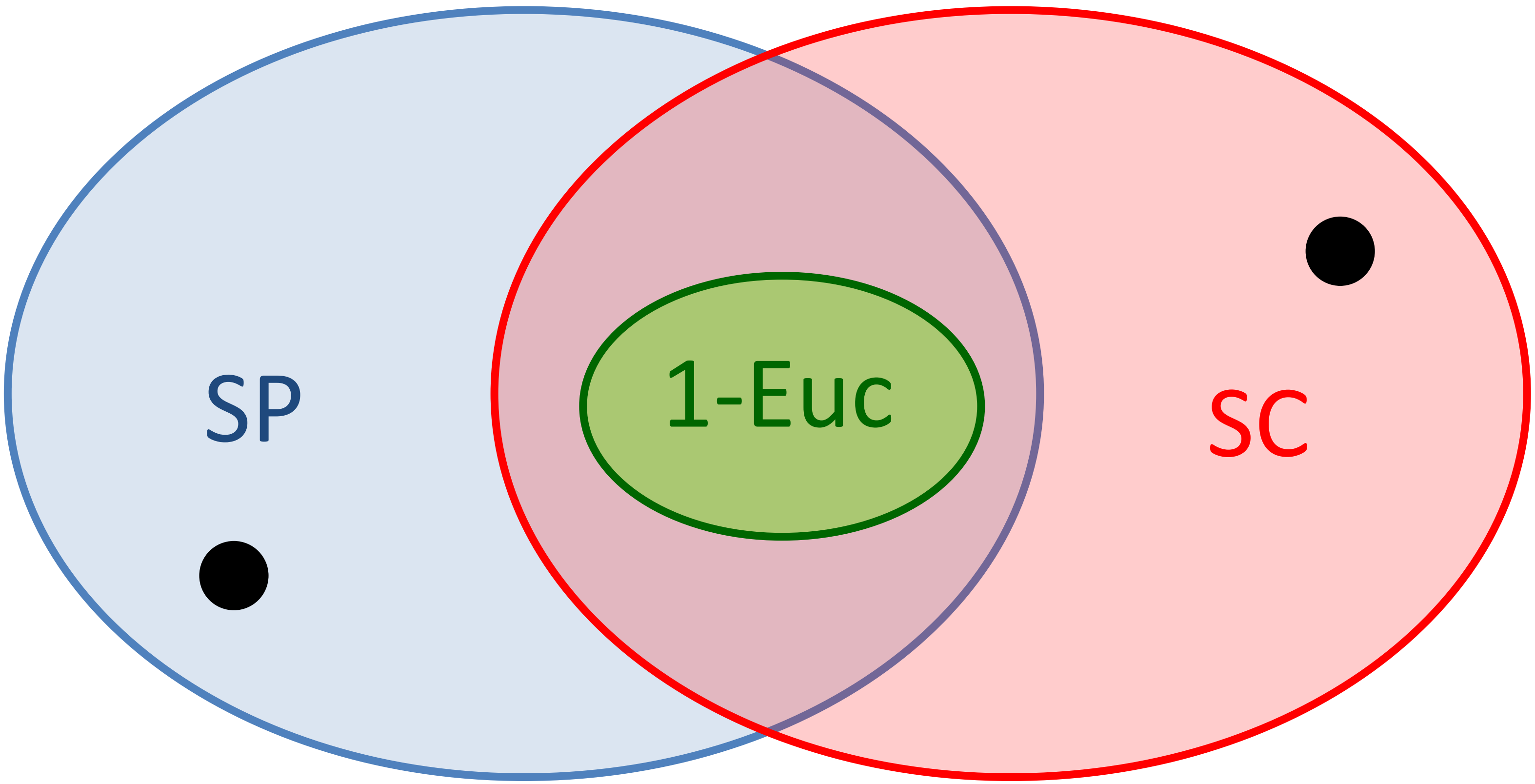
1-Euc = SP $\cap$ SC?

- Observation: There exists a preference profile that is SP and SC, but not 1-Euclidean

v_1: B C D E A F
v_2: D E C B A F
v_3: D E F C B A



- SC wrt $v_1 < v_2 < v_3$, SP wrt $A < B < C < D < E < F$
- Not 1-Euclidean:
  - $(x(A) + x(E))/2 < x(v_1) < (x(B) + x(C))/2$
  - $(x(C) + x(D))/2 < x(v_2) < (x(A) + x(F))/2$
  - $(x(B) + x(F))/2 < x(v_3) < (x(D) + x(E))/2$


SP
1-Euc
SC